# Errata

of the book

# 3+1 Formalism in General Relativity

https://relativite.obspm.fr/3p1

7 March 2024

*Many thanks to Ngô Quôc Anh, Zu-Cheng Chen, F. Semih Dündar, Ted Jacobson, Yat-lo Lay, Jorge Lopes, Grégoire Martinon, Christian Schell and Claire Somé!*

---

- Page 12, Eq. (2.23): the symbol $T$ in the r.h.s. should not be bold, i.e. the equation should be
$$\boldsymbol{T} = T^{\alpha_1\ldots\alpha_k}{}_{\beta_1\ldots\beta_\ell}\, \boldsymbol{e}_{\alpha_1}\otimes\ldots\otimes\boldsymbol{e}_{\alpha_k}\otimes\boldsymbol{e}^{\beta_1}\otimes\ldots\otimes\boldsymbol{e}^{\beta_\ell}$$
- Page 16, 4th line: the symbol $u^\alpha$ should not be bold.
- Page 20, Eq. (2.56): the vector $\boldsymbol{u}$ in the r.h.s. must be replaced by $\boldsymbol{v}$, so that the equation becomes
$$\boldsymbol{\nabla}_{\boldsymbol{v}}\boldsymbol{T} := \boldsymbol{\nabla}\boldsymbol{T}(\underbrace{.,\ldots,.}_{k+\ell \text{ slots}},\boldsymbol{v}).$$
- Page 20, Eq. (2.58): the symbol $T$ in the r.h.s. should not be bold, i.e. the equation should be
$$(\boldsymbol{\nabla}\cdot\boldsymbol{T})^{\alpha_1\ldots\alpha_{k-1}}{}_{\beta_1\ldots\beta_\ell} = \boldsymbol{\nabla}_\mu T^{\alpha_1\ldots\alpha_{k-1}\mu}{}_{\beta_1\ldots\beta_\ell}.$$
- Page 24, Eq. (2.81): a factor $1/2$ is missing in the r.h.s.: the equation should be
$$R^\gamma{}_{\delta\alpha\beta} = \frac{R}{2}\left(\delta^\gamma{}_\alpha\, g_{\delta\beta} - \delta^\gamma{}_\beta\, g_{\delta\alpha}\right) \qquad (n=2).$$
- Page 31, in the second line, just after "In other words": insert "for any $p\in\Sigma$,".
- Pages 31, replace any occurrence of $\mathscr{T}_p(\hat{\Sigma})$ by $\mathscr{T}_{\Phi^{-1}(p)}(\hat{\Sigma})$, as well as any occurrence of $\mathscr{T}^*_p(\hat{\Sigma})$ by $\mathscr{T}^*_{\Phi^{-1}(p)}(\hat{\Sigma})$.
- Page 50, in the left-hand sides of the two unnumbered equations at the end of the page, as well as in the text two lines above them: the term $D_\beta D_\gamma v^\gamma$ must be replaced by $D_\beta D_\alpha v^\gamma$, so that these left-hand sides becomes
$$D_\alpha D_\beta v^\gamma - D_\beta D_\alpha v^\gamma = \cdots$$

- Page 33, Eq. (3.11): insert a $\pm$ sign immediately after the equal sign, so that Eq. (3.11) becomes

$$\boldsymbol{n} := \pm \left( \pm \overrightarrow{\boldsymbol{\nabla}} t \cdot \overrightarrow{\boldsymbol{\nabla}} t \right)^{-1/2} \overrightarrow{\boldsymbol{\nabla}} t .$$

- Page 33, first line below Eq. (3.13): after "the other one being $\boldsymbol{n'} = -\boldsymbol{n}$.", add "The choice made in Eq. (3.11) ensures that $\boldsymbol{n}$ is directed towards increasing values of $t$".

- Page 35, first line below the unnumbered equation at the top of the page: replace $\alpha := (-\overrightarrow{\boldsymbol{\nabla}} t \cdot \overrightarrow{\boldsymbol{\nabla}} t)^{-1/2}$ by $\alpha := -(-\overrightarrow{\boldsymbol{\nabla}} t \cdot \overrightarrow{\boldsymbol{\nabla}} t)^{-1/2}$.

- Page 51, unnumbered equation between Eqs. (3.74) and (3.75): some indices of the Riemann tensor are inline instead of being subscripts; the correct equation should be written

$$\begin{aligned} \gamma^{\alpha\beta} \gamma_{\alpha\mu} n^\nu \gamma^\rho{}_\beta n^\sigma {}^4R^\mu{}_{\nu\rho\sigma} &= \gamma^\rho{}_\mu n^\nu n^\sigma {}^4R^\mu{}_{\nu\rho\sigma} = \underbrace{{}^4R^\mu{}_{\nu\mu\sigma}}_{{}^4R_{\nu\sigma}} n^\nu n^\sigma + \underbrace{{}^4R^\mu{}_{\nu\rho\sigma} n^\rho n_\mu n^\nu n^\sigma}_{0} \\ &= {}^4R_{\mu\nu} n^\mu n^\nu . \end{aligned}$$

- Page 62, second line in the formula for $a_\alpha$ at the page's top: some signs are not correct; this line should be

$$= \frac{1}{N} n_\alpha n^\mu \nabla_\mu N - N n^\mu \nabla_\alpha \left( -\frac{1}{N} n_\mu \right) = \frac{1}{N} n_\alpha n^\mu \nabla_\mu N - \frac{1}{N} \nabla_\alpha N \underbrace{n^\mu n_\mu}_{-1} + \underbrace{n^\mu \nabla_\alpha n_\mu}_{0}$$

- Page 63, 3 lines below Eq. (4.22): $b^2 N$ must be replaced by $b^3 N$, so that the equation becomes $n^\mu \nabla_\mu N = (x^2 + y^2 + z^2)/(b^3 N)$.

- Page 86, Eq. (5.65): two terms $\beta^k$ are mistyped as $\beta^K$; the equation should be

$$\mathscr{L}_{\boldsymbol{\beta}} K_{ij} = \beta^k \frac{\partial K_{ij}}{\partial x^k} + K_{kj} \frac{\partial \beta^k}{\partial x^i} + K_{ik} \frac{\partial \beta^k}{\partial x^j} .$$

- Page 87, Eq. (5.76): the final $\beta^k$ is mistyped as $\beta^K$; the equation should be

$$\mathscr{L}_{\boldsymbol{\beta}} K_{ij} = \beta^k \frac{\partial K_{ij}}{\partial x^k} + K_{kj} \frac{\partial \beta^k}{\partial x^i} + K_{ik} \frac{\partial \beta^k}{\partial x^j}$$

- Page 129, Eq. (6.134b): the first factor $1/\mu_0$ should be $1/(2\mu_0)$ and the term $U_K$ should be replaced by $U_k$, so that the equation becomes

$$\begin{aligned} \mathscr{F}_i^j := \sqrt{\gamma} \Bigg\{ & N \left[ P + \frac{1}{2\mu_0} \left( \frac{B_k B^k}{\Gamma^2} + (U_k B^k)^2 \right) \right] \delta^j{}_i + \frac{B_k B^k}{\mu_0} V_i U^j \\ & + \left[ (E+P) U_i - \frac{U_k B^k}{\mu_0} B_i \right] V^j - \frac{N}{\mu_0} \left[ \frac{B_i}{\Gamma^2} - U_k B^k U_i \right] B^j \Bigg\}, \\ & 1 \le i \le 3 \end{aligned}$$

- Page 153, 3 lines above Eq. (7.100): the phrase "the connection $\tilde{\boldsymbol{D}}$ is simply $\tilde{\boldsymbol{D}}$" must be replaced by "the connection $\tilde{\boldsymbol{D}}$ is simply $\bar{\boldsymbol{D}}$"

- Page 162, Eq. (8.7): the term $\beta_i$ must be replaced by $\beta^i$, so that the equation becomes

$$H = -\int_{\Sigma_t^{\mathrm{int}}} \left(NC_0 - 2\beta^i C_i\right) \sqrt{\gamma}\, \mathrm{d}^3x - 2\oint_{\mathscr{S}_t} \left[N(\kappa - \kappa_0) + \beta^i (K_{ij} - K\gamma_{ij}) s^j\right] \sqrt{q}\, \mathrm{d}^2y.$$

- Page 165, Eq. (8.17b): the term $1/\eta$ must be replaced by $1/r$, so that the equation becomes

$$\bar{\Gamma}^{\theta}{}_{r\theta} = \bar{\Gamma}^{\theta}{}_{\theta r} = \frac{1}{r} \quad \text{and} \quad \bar{\Gamma}^{\theta}{}_{\varphi\varphi} = -\cos\theta \sin\theta$$

- Page 178, unnumbered equation below Eq. (8.61): the index $\nu$ in ${}^4R^{\mu}{}_{\nu}$ is ill placed; the equation should be

$$\nabla_\nu \nabla^\mu k^\nu - \nabla^\mu \underbrace{\nabla_\nu k^\nu}_{0} = {}^4R^{\mu}{}_{\nu} k^\nu.$$

- Page 184: the grey box delimiting Example 8.7 should terminate just below Eq. (8.80), i.e. the text starting by "Let us now find..." should typed be on a white background.

- Page 187, Eq. (9.2): the indices $i$ and $j$ of $K^i{}_j$ must be swapped, so that the equation becomes

$$\boxed{D_j K^j{}_i - D_i K = 8\pi p_i}.$$

- Page 191, unnumbered equation at the top of the page: the term $\hat{A}^{jl}$ must be replaced by $(LX)^{jl}$, so that the equation becomes

$$\int_{\Sigma_0} \tilde{\gamma}_{ij} C^i \tilde{D}_k \hat{A}^{jk} \sqrt{\tilde{\gamma}}\, \mathrm{d}^3x = -\frac{1}{2} \int_{\Sigma_0} \tilde{\gamma}_{ij} \tilde{\gamma}_{kl} (\tilde{L}C)^{ik} (LX)^{jl} \sqrt{\tilde{\gamma}}\, \mathrm{d}^3x.$$

- Page 193, 3 lines above Section 8.2.4: replace "that it always solvable" by "that it is always solvable".

- Page 203, 2 lines above Eq. (9.75): replace the second occurrence of "(9.66)" by "(8.44)".

- Page 205, 4 lines before the end of Remark 9.4: replace "the thin sandwich system does have" by "the thin sandwich system does not have".

- Page 206, unnumbered equation between Eqs. (9.85) and (9.86): the plus sign in the r.h.s. should be minus, so that the equation should be

$$\tilde{D}_i \tilde{D}^i N + 2\tilde{D}_i \ln\Psi\, \tilde{D}^i N = \Psi^{-1} \left[\tilde{D}_i \tilde{D}^i (N\Psi) - N \tilde{D}_i \tilde{D}^i \Psi\right].$$

- Page 215, 5th line: replace "boosted boosted" by "boosted".

- Page 225, last line: replace $\Sigma_{\partial t}$ by $\Sigma_{\delta t}$.

- Page 233, first line above Eq. (10.23): replace "in the form (10.23)" by "in the form (10.19)".
- Page 237, first line below Eq. (10.33): replace $\Sigma_{t+\partial t}$ by $\Sigma_{t+\delta t}$.
- Page 243, Eq. (10.56): replace $==$ by $=$.
- Page 244, first line below Eq. (10.59): replace "$K$ is a positive function" by "$k$ is a positive function".
- Page 245, first line of Remark 10.10: replace "where $K$" by "where $k$".