Figures of lecture 1

Definition and main properties of black holes

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https://relativite.obspm.fr/blackholes/aei23/

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Potsdam, Germany
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It all started 50 years ago at Les Houches...

The Four Laws of Black Hole Mechanics

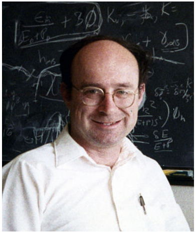
J. M. Bardeen*
Department of Physics, Yale University, New Haven, Connecticut, USA

B. Carter and S. W. Hawking
Institute of Astronomy, University of Cambridge, England

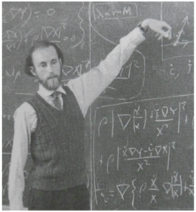
Received January 24, 1973

This work was carried out while the authors were attending the 1972 Les Houches Summer School on Black Holes. The authors would like to thank Larry Smarr, Bryce de Witt and other participants of the school for valuable discussions.

[Commun. Math. Phys., **31**, 161 (1973)]



James Bardeen, ~1980



Brandon Carter, 1972

Stephen Hawking, 1972

Picture credit: 1: Wikipedia
2, 3: Kip Thorne

Also at Les Houches 1972: the first Kerr BH shadow...

... presented in J. Bardeen's lecture:

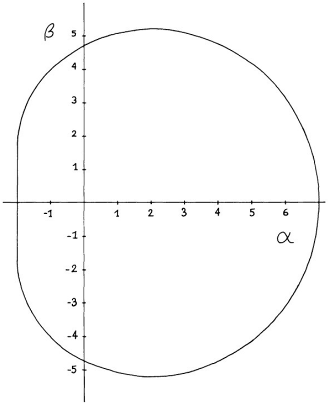


Figure 6. The apparent shape of an extreme ($a = m$) Kerr black hole as seen by a distant observer in the equatorial plane, if the black hole is in front of a source of illumination with an angular size larger than that of the black hole.

[J. Bardeen, in *Black Holes – Les astres occlus*, proc. of *Les Houches Summer School 1972*, ed. C. DeWitt and B. DeWitt, (1973), p. 215.]

These lectures

provide an introduction to BH thermodynamics

- focussing on classical (non-quantum) aspects
- keeping the spacetime dimension n general
- not restricting the theory of gravity to general relativity

Home page

https://relativite.obspm.fr/blackholes/aei23

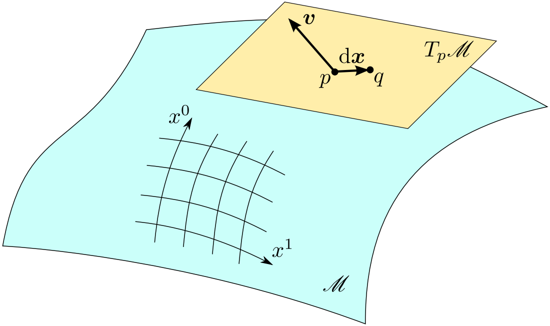
includes

- the lecture notes (draft)
- some SageMath notebooks
- these slides

Framework

spacetime = $(\mathscr{M}, \boldsymbol{g})$

- $\mathscr{M}$: n-dimensional smooth manifold
- $\boldsymbol{g}$: Lorentzian metric on $\mathscr{M}$

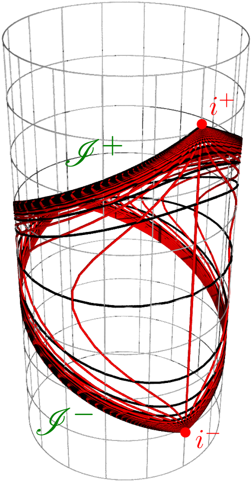


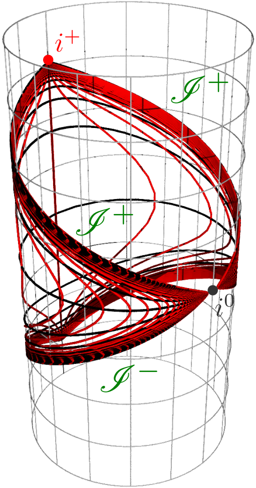
Smooth manifold: topological space $\mathscr{M}$ that *locally* resembles $\mathbb{R}^n$ (but maybe not globally)
$\Longrightarrow$ **coordinate charts**
$\Longrightarrow$ **tangent vectors**

Remark: vector connecting two points p and q defined only for p and q infinitely close

Conformal completion of Minkowski spacetime

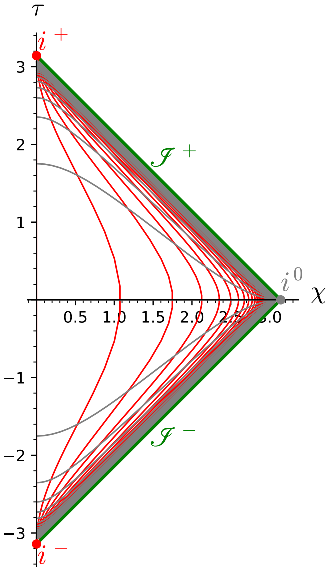
Embedding into the Einstein cylinder





- on $\mathscr{E}$:
  $-\infty < \tau < +\infty$
  $0 \leq \chi \leq \pi$
- on $\mathscr{M}$:
  $\chi - \pi < \tau < \pi - \chi$
  $0 \leq \chi < \pi$

cf. https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/conformal_Minkowski.ipynb for an interactive 3D view



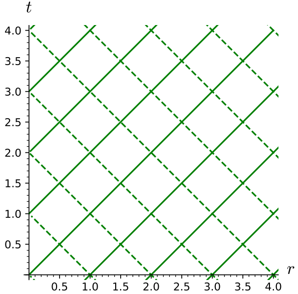
View in the (τ, χ) coordinate plane

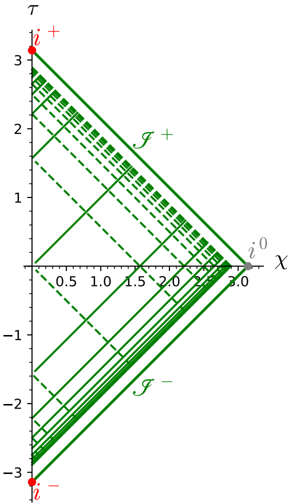
$0 \leq \chi < \pi$
$\chi - \pi < \tau < \pi - \chi$

red: $r = \text{const}$
grey: $t = \text{const}$

Conformal completion of Minkowski spacetime

Conformal diagram





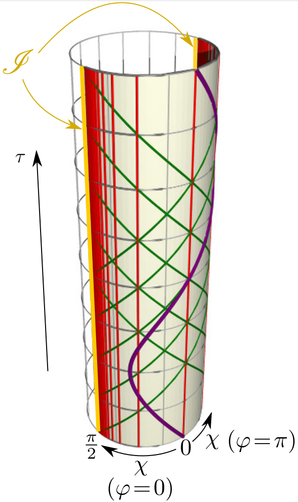
Radial null geodesics:
solid:
$u := t - r = \text{const}$
dashed:
$v := t + r = \text{const}$

Radial null geodesics appear as straight lines with $\pm 45°$ slope

$\Longrightarrow$ conformal diag. also called Penrose diagram or Carter-Penrose diagram

Conformal completion of anti-de Sitter spacetime

Embedding into the Einstein cylinder



- on $\mathscr{E}$:

  $-\infty < \tau < +\infty$

  $0 \leq \chi \leq \pi$

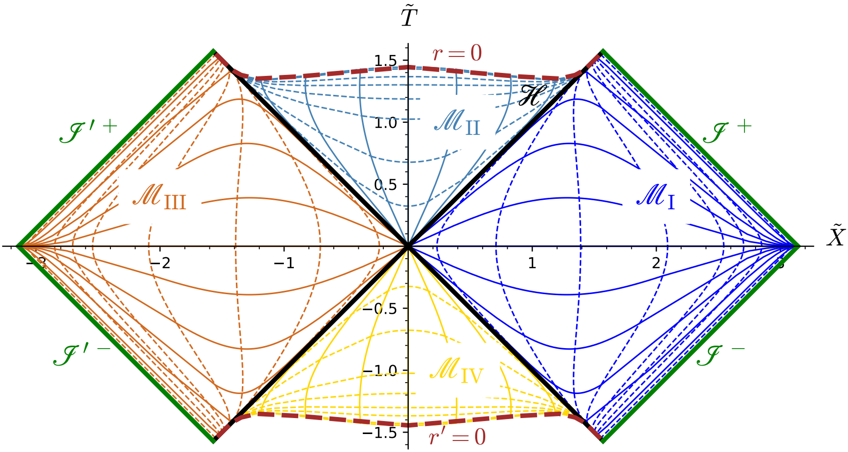
- on $\mathscr{M}$:

  $-\infty < \tau < +\infty$

  $0 \leq \chi < \pi/2$

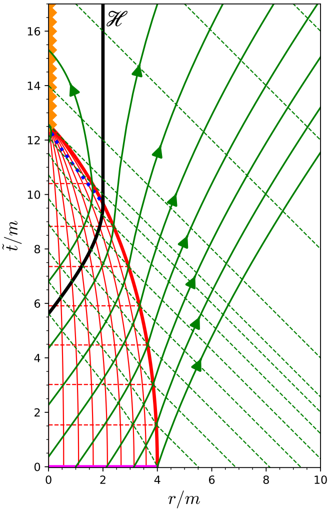
cf. https://nbviewer.org/github/sagemanifolds/SageManifolds/blob/master/Notebooks/SM_anti_de_Sitter.ipynb for an interactive 3D view

Example 1: the Schwarzschild black hole

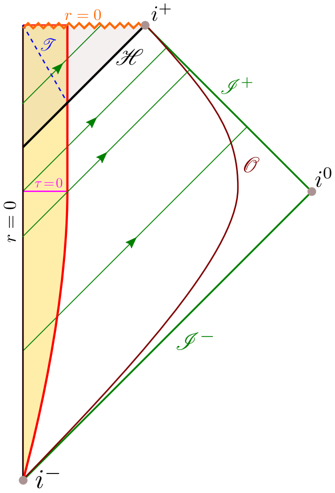


Maximal extension of Schwarzschild spacetime represented in Penrose-Frolov-Novikov coordinates

Example 2: black hole in Oppenheimer-Snyder collapse



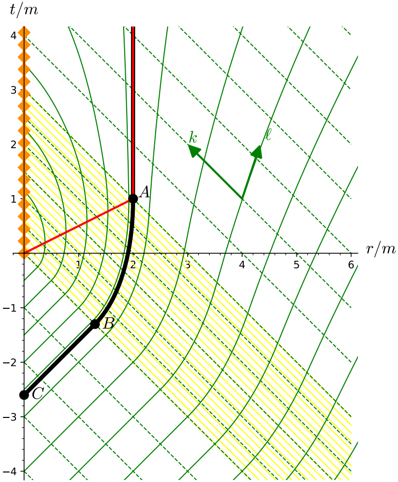
Eddington-Finkelstein coordinates



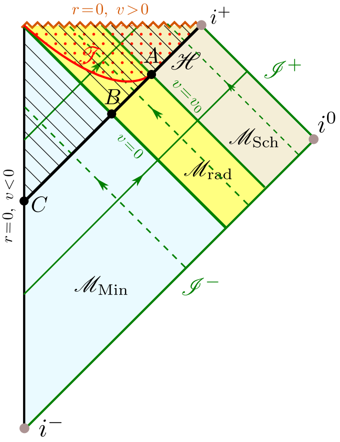
Carter-Penrose diagram

https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Oppenheimer_Snyder.ipynb

Example 3: black hole in Vaidya collapse



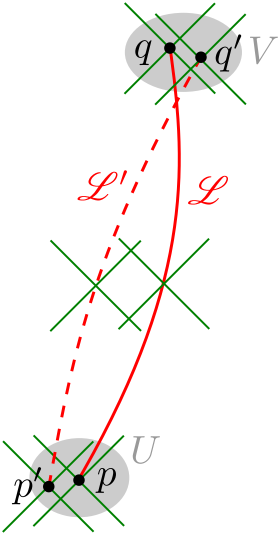
Eddington-Finkelstein coordinates



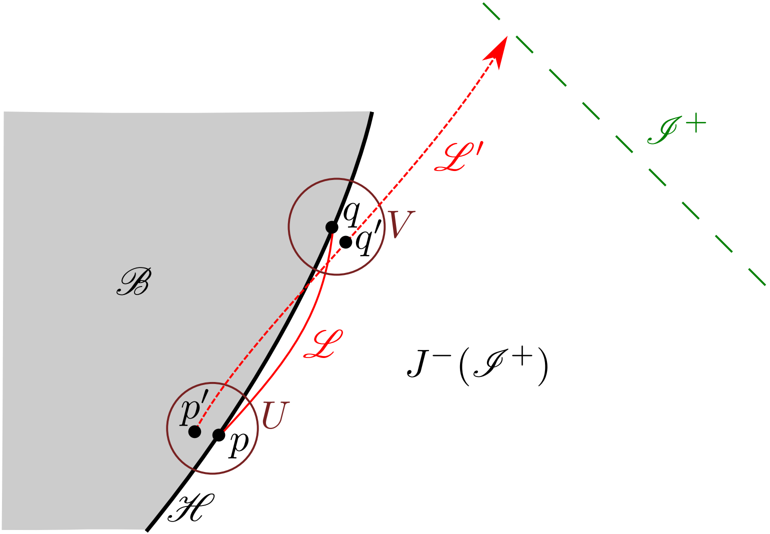
Carter-Penrose diagram

https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/Vaidya.ipynb

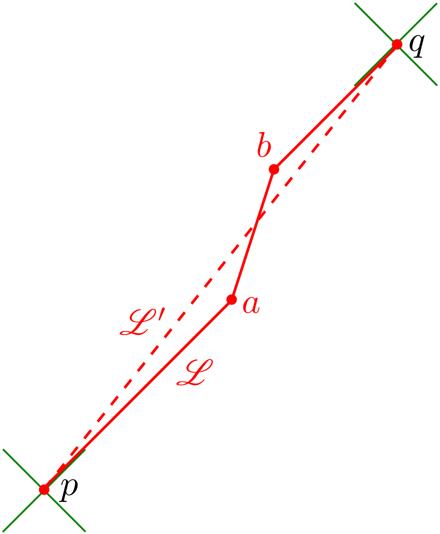
Lemma: stability of timelike curves



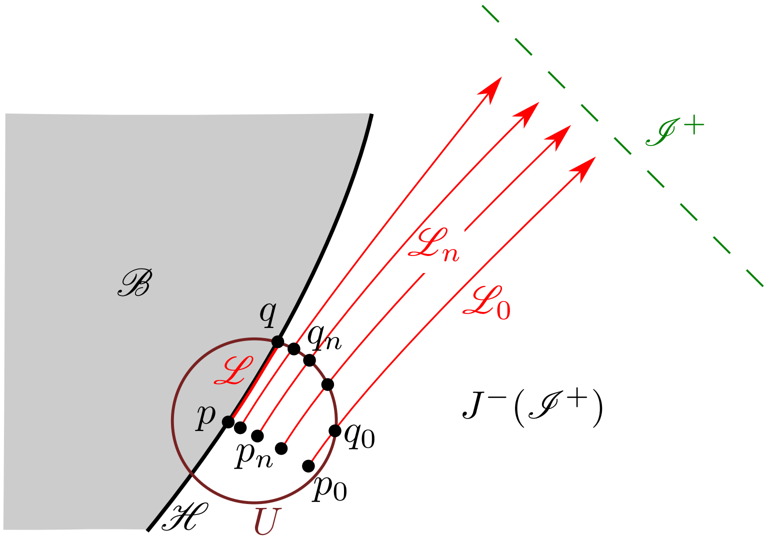
Proving that $\mathscr{H}$ is achronal



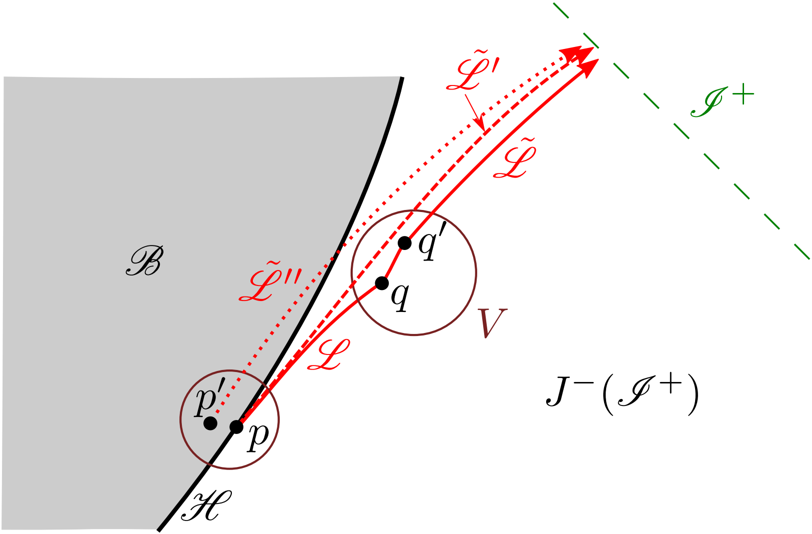
The timelike segment lemma



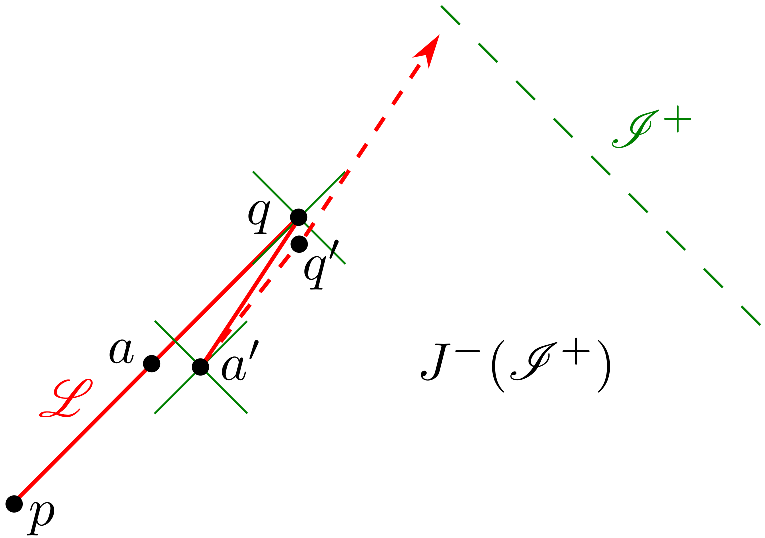
Causal curve connecting p to q



Proving by contradiction that q lies in $\mathscr{H}$

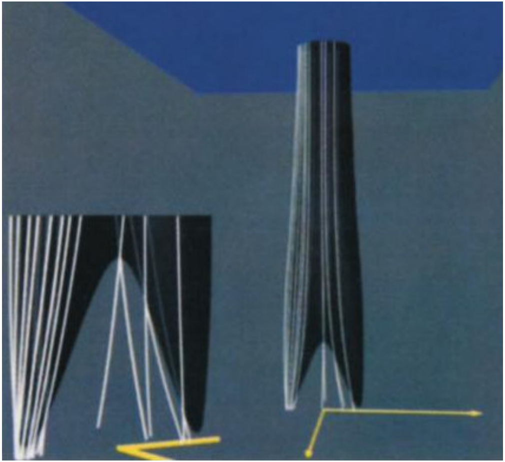


Proving that $\mathscr{L}$ lies entirely in $\mathscr{H}$



Event horizon of a binary black hole merger

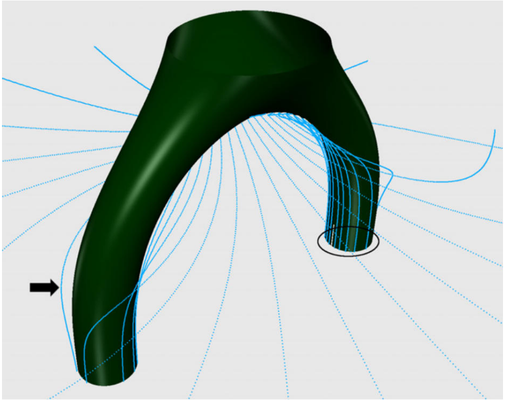
Head-on merger



[R.A. Matzner et al., Science **270**, 941 (1995)]

Event horizon of a binary black hole merger

Head-on merger

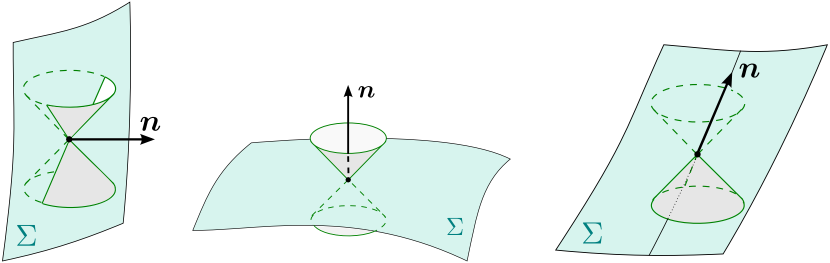


[Cohen, Pfeiffer & Scheel, CQG **26**, 035005 (2009)]

Three kinds of hypersurfaces

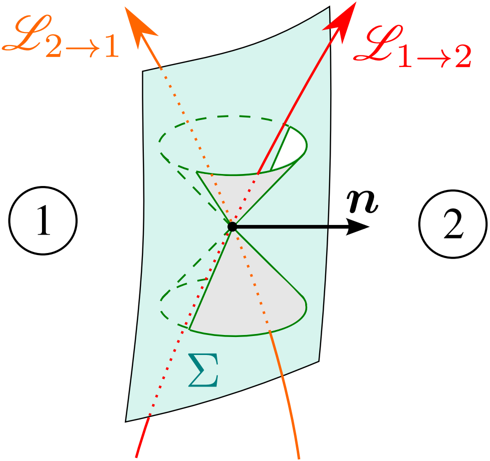
Boundary in spacetime $\Longrightarrow$ $(n-1)$-dimensional submanifold, i.e. **hypersurface**

Locally, a hypersurface Σ can be of one of 3 types:



Σ timelike	Σ spacelike	Σ null
$\boldsymbol{g}\|_\Sigma$ Lorentzian	$\boldsymbol{g}\|_\Sigma$ Riemannian	$\boldsymbol{g}\|_\Sigma$ degenerate
$\boldsymbol{n}$ spacelike	$\boldsymbol{n}$ timelike	$\boldsymbol{n}$ null (and tangent to Σ)

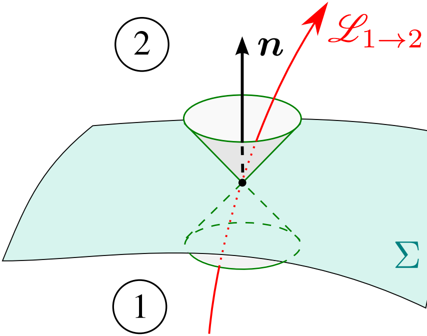
Timelike hypersurface



For worldlines $\mathscr{L}$ directed towards the future:

timelike hypersurface = 2-way membrane
$\Longrightarrow$ not eligible for a black hole boundary

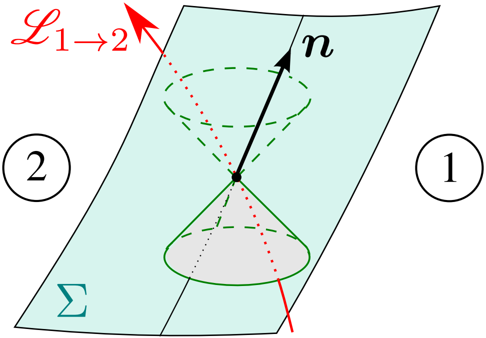
Spacelike hypersurface



For worldlines $\mathscr{L}$ directed towards the future:

spacelike hypersurface = 1-way membrane
$\Longrightarrow$ eligible for a black hole boundary

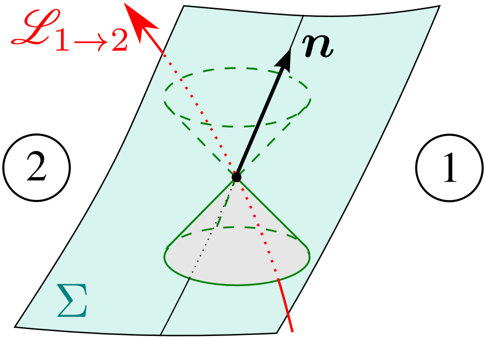
Null hypersurface



For worldlines $\mathscr{L}$ directed towards the future:

null hypersurface = 1-way membrane
$\Longrightarrow$ eligible for a black hole boundary...

Null hypersurface



For worldlines $\mathscr{L}$ directed towards the future:

null hypersurface = 1-way membrane
$\Longrightarrow$ eligible for a black hole boundary...
...and elected!

Theorem (Penrose)

The event horizon of a black hole is a topological hypersurface of spacetime. Wherever it is smooth, it is a null hypersurface.