Figures of lecture 1

Definition and main properties of black holes

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https://relativite.obspm.fr/blackholes/paris23/

PSL graduate programs in Physics and in Astrophysics
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Home page for the lectures

https://relativite.obspm.fr/blackholes/paris23

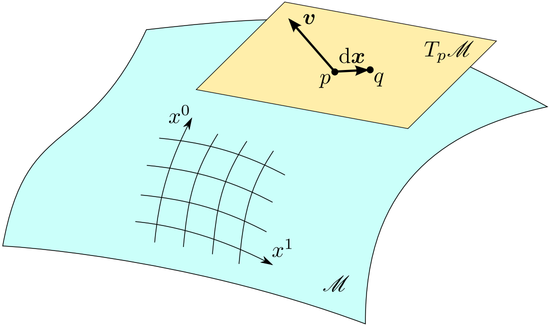
includes

- the lecture notes (draft)
- some SageMath notebooks
- these slides

Spacetime

spacetime $= (\mathscr{M}, \boldsymbol{g})$

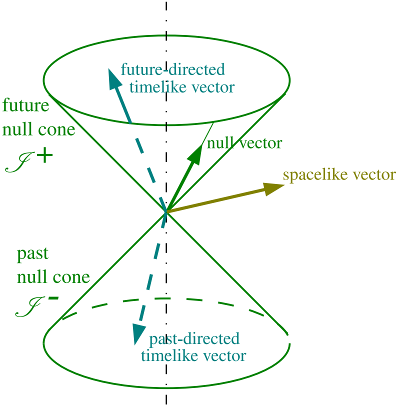
- $\mathscr{M}$: n-dimensional smooth manifold
- $\boldsymbol{g}$: Lorentzian metric on $\mathscr{M}$



Smooth manifold: topological space $\mathscr{M}$ that *locally* resembles $\mathbb{R}^n$ (but maybe not globally)
$\Longrightarrow$ **coordinate charts**
$\Longrightarrow$ **tangent vectors**

Remark: vector connecting two points p and q defined only for p and q infinitely close

Metric's null cone

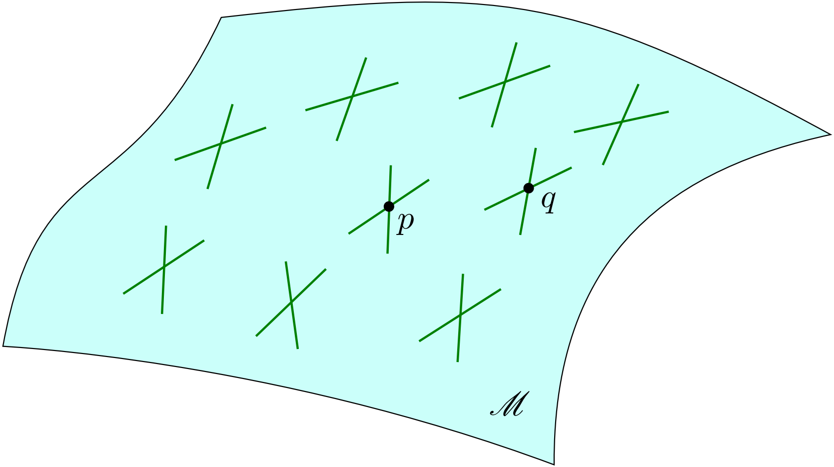


Vector $\boldsymbol{v} \in T_p\mathscr{M}$ is

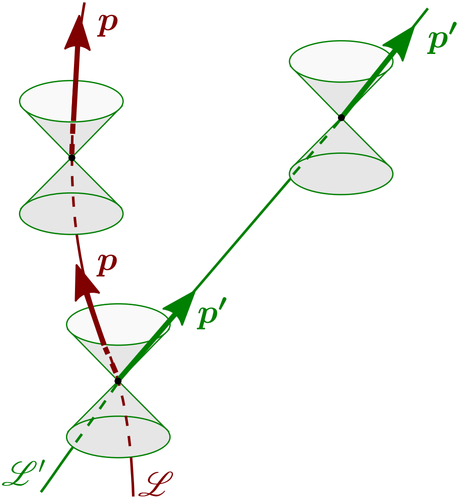
- **spacelike** $\iff \boldsymbol{g}(\boldsymbol{v}, \boldsymbol{v}) > 0$
- **null** $\iff \boldsymbol{g}(\boldsymbol{v}, \boldsymbol{v}) = 0$
- **timelike** $\iff \boldsymbol{g}(\boldsymbol{v}, \boldsymbol{v}) < 0$

Additional assumption:
the spacetime $(\mathscr{M}, \boldsymbol{g})$ is **time-oriented**
$\Longrightarrow$ future and past directions continuously defined over all $\mathscr{M}$

Lorentzian manifold $(\mathscr{M}, \boldsymbol{g})$



Worldlines



Particle described by its spacetime extent: **worldline** $\mathscr{L}$

massive part. $\iff$ timelike worldline
massless part. $\iff$ null worldline
(tachyon $\iff$ spacelike worldline)

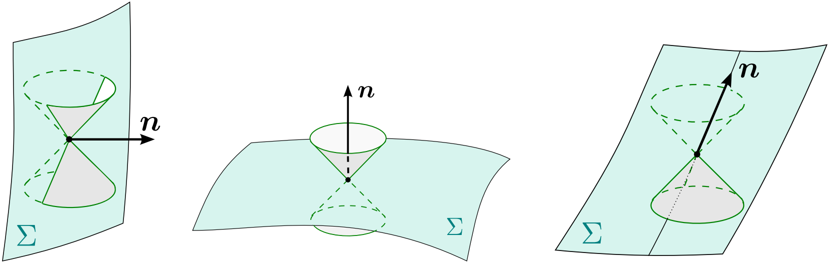
Dynamics of a *simple* particle (no spin, no internal structure) entirely described by a future-directed vector field tangent to the worldline: the **energy-momentum** $\boldsymbol{p}$

Particle's mass: $m = \sqrt{-\boldsymbol{g}(\boldsymbol{p}, \boldsymbol{p})}$

Three kinds of hypersurfaces

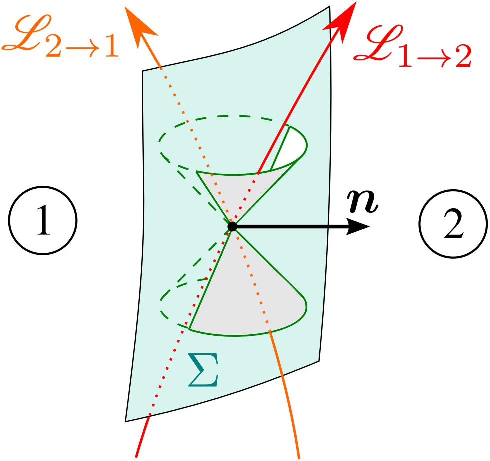
Boundary in spacetime $\Longrightarrow$ $(n-1)$-dimensional submanifold, i.e. **hypersurface**

Locally, a hypersurface Σ can be of one of 3 types:



Σ timelike	Σ spacelike	Σ null
$\boldsymbol{g}\vert_\Sigma$ Lorentzian	$\boldsymbol{g}\vert_\Sigma$ Riemannian	$\boldsymbol{g}\vert_\Sigma$ degenerate
$\boldsymbol{n}$ spacelike	$\boldsymbol{n}$ timelike	$\boldsymbol{n}$ null (and tangent to Σ)

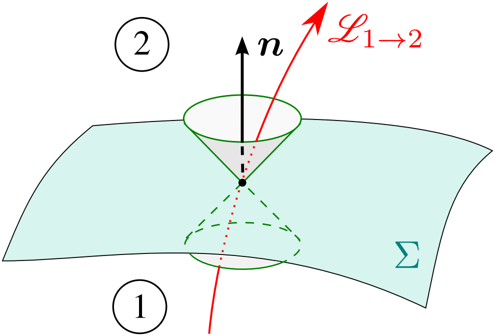
Timelike hypersurface



For worldlines $\mathscr{L}$ directed towards the future:

timelike hypersurface = 2-way membrane
$\Longrightarrow$ not eligible for a black hole boundary

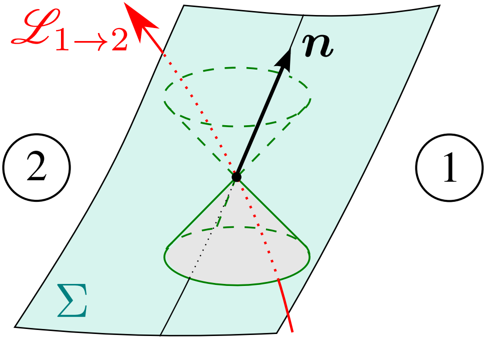
Spacelike hypersurface



For worldlines $\mathscr{L}$ directed towards the future:

spacelike hypersurface = 1-way membrane
$\Longrightarrow$ eligible for a black hole boundary

Null hypersurface

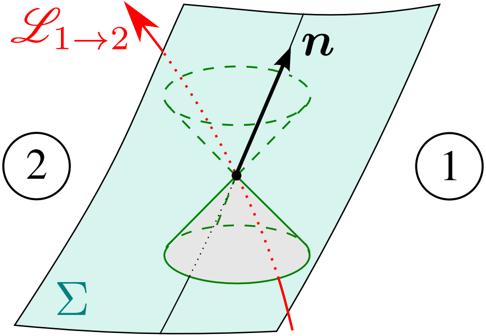


For worldlines $\mathscr{L}$ directed towards the future:

null hypersurface = 1-way membrane

$\Longrightarrow$ eligible for a black hole boundary...

Null hypersurface



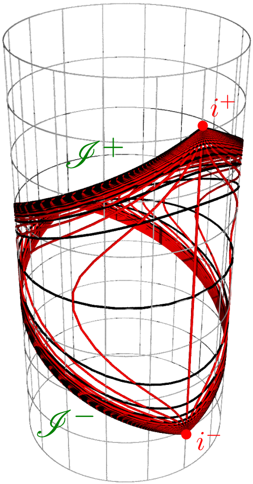
For worldlines $\mathscr{L}$ directed towards the future:

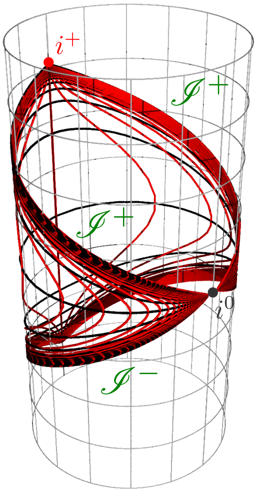
null hypersurface = 1-way membrane
$\Longrightarrow$ eligible for a black hole boundary...
...and elected! (as a consequence of the formal definition of a black hole, to be given later)

The event horizon of a black hole is a topological hypersurface of spacetime. Wherever it is smooth, it is a null hypersurface.

Conformal completion of Minkowski spacetime

Embedding into the Einstein cylinder



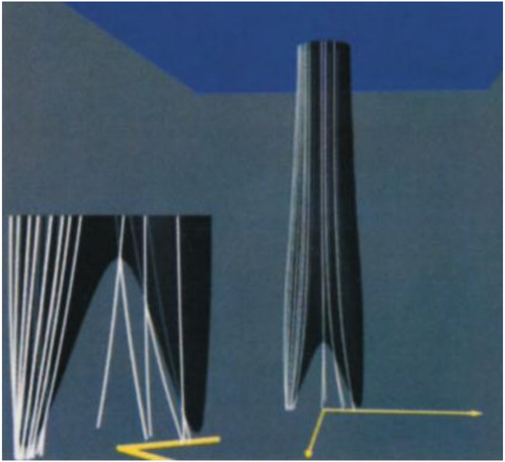


- on $\mathscr{E}$:
  $-\infty < \tau < +\infty$
  $0 \leq \chi \leq \pi$
- on $\mathscr{M}$:
  $\chi - \pi < \tau < \pi - \chi$
  $0 \leq \chi < \pi$

cf. https://nbviewer.org/github/egourgoulhon/BHLectures/blob/master/sage/conformal_Minkowski.ipynb for an interactive 3D view

Event horizon of a binary black hole merger

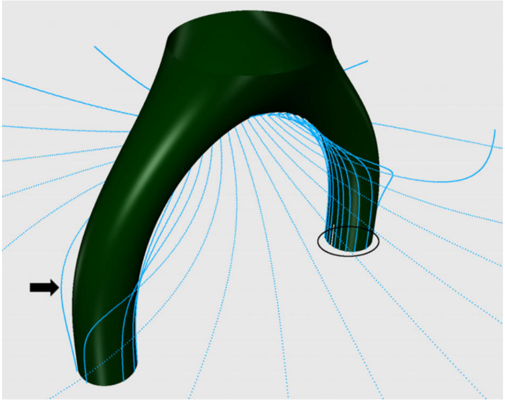
Head-on merger



[R.A. Matzner et al., Science **270**, 941 (1995)]

Event horizon of a binary black hole merger

Head-on merger



[Cohen, Pfeiffer & Scheel, CQG **26**, 035005 (2009)]